PacRim-2019

Dynamic Games in Federated Learning Training

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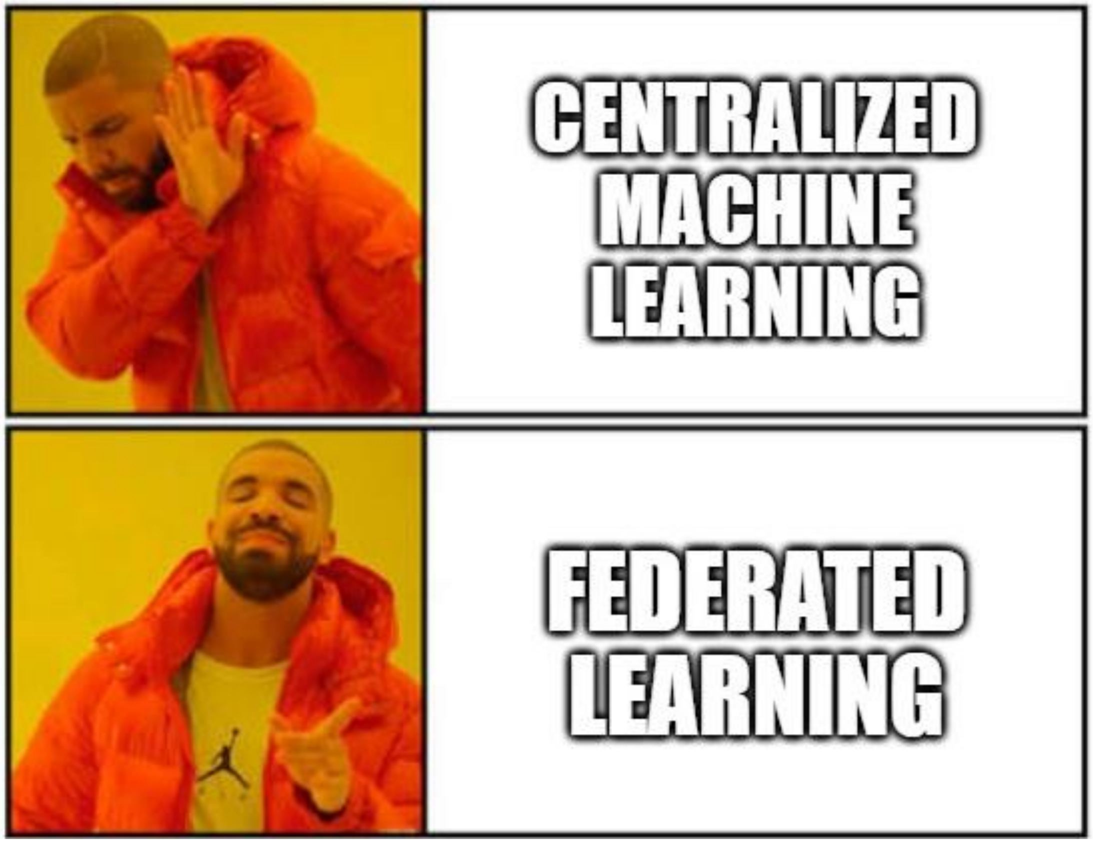
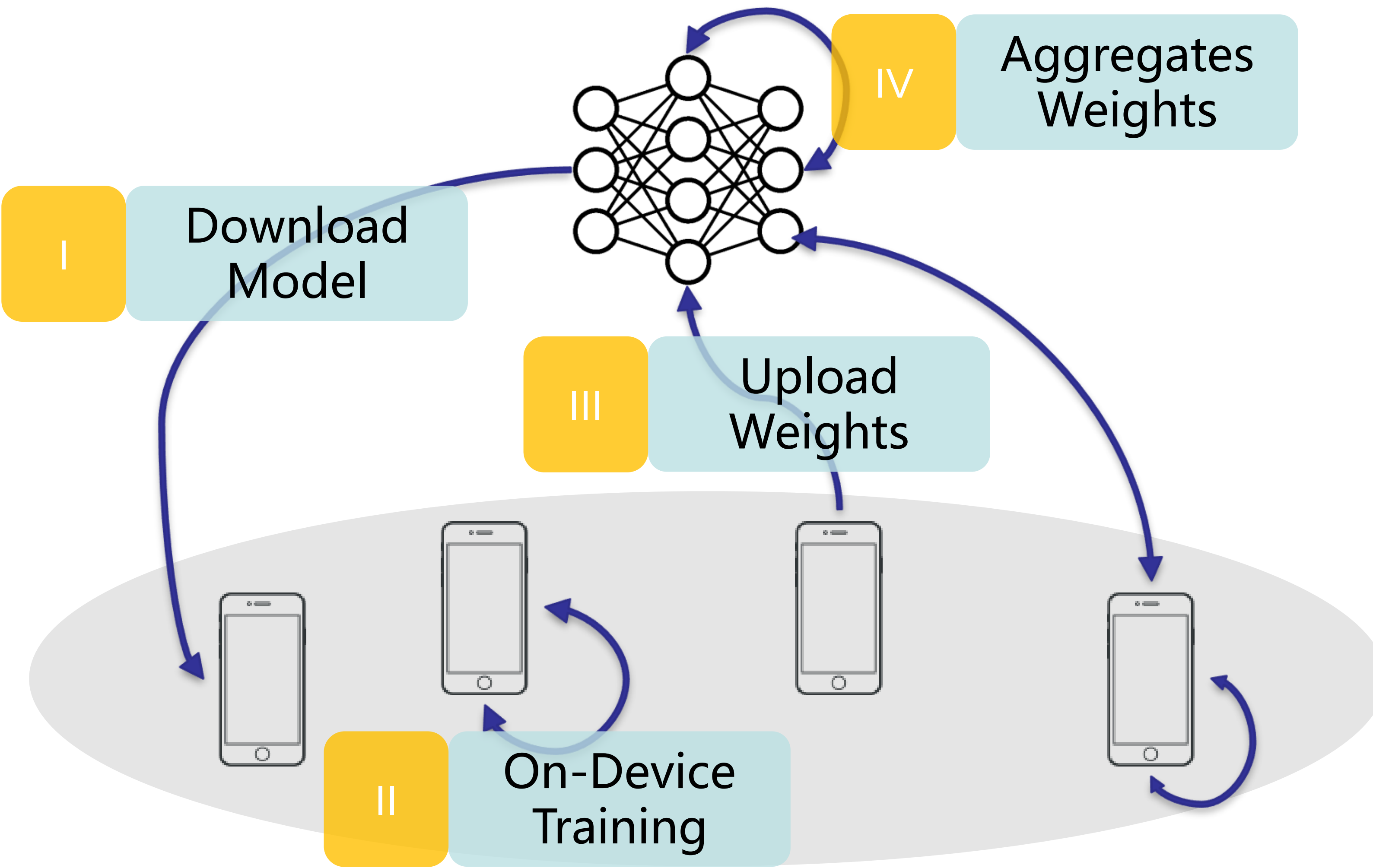
- System Description
- Dynamic Games Formulation
- Equilibrium Analysis
- Numerical Evaluation

Introduction



Introduction



System Description

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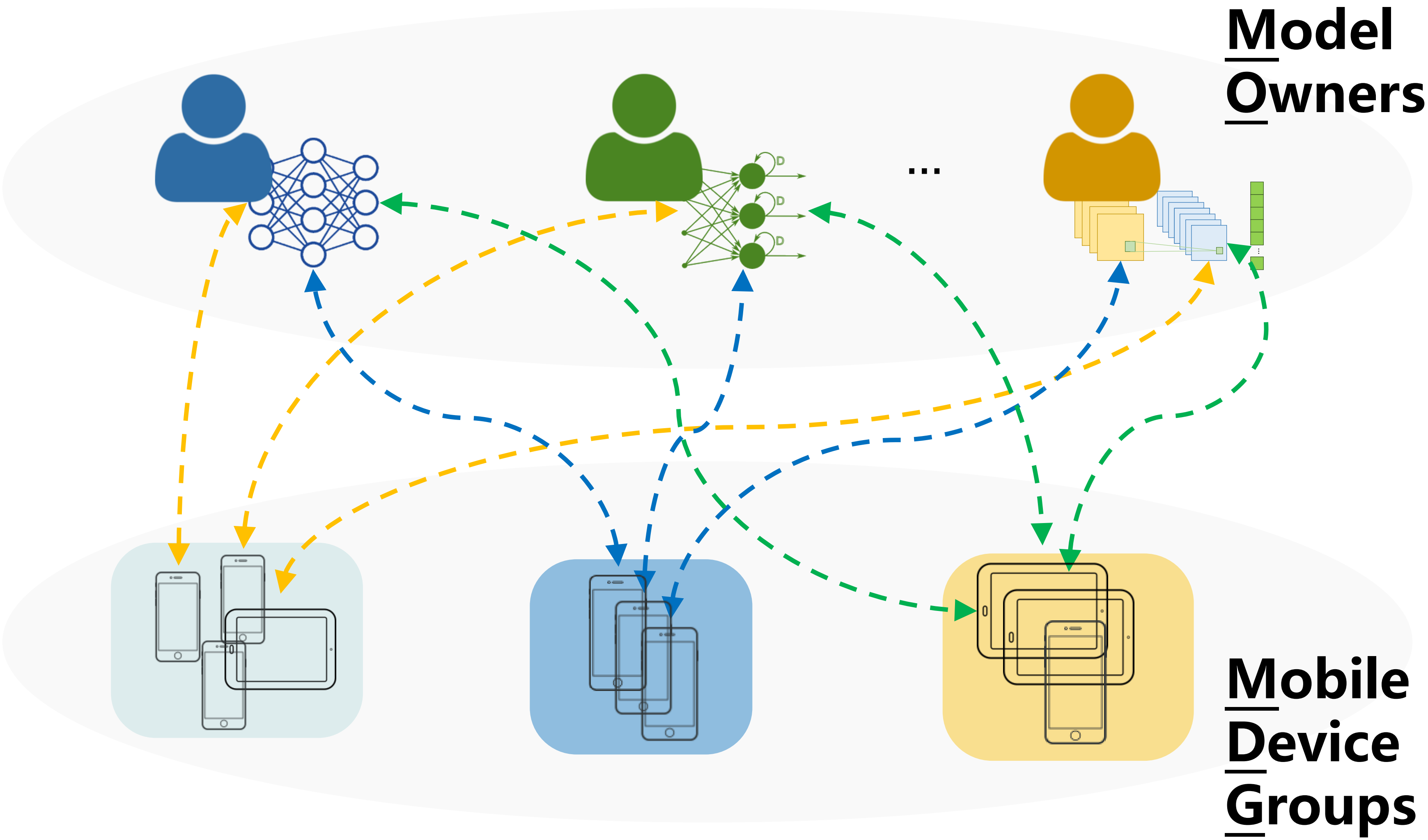
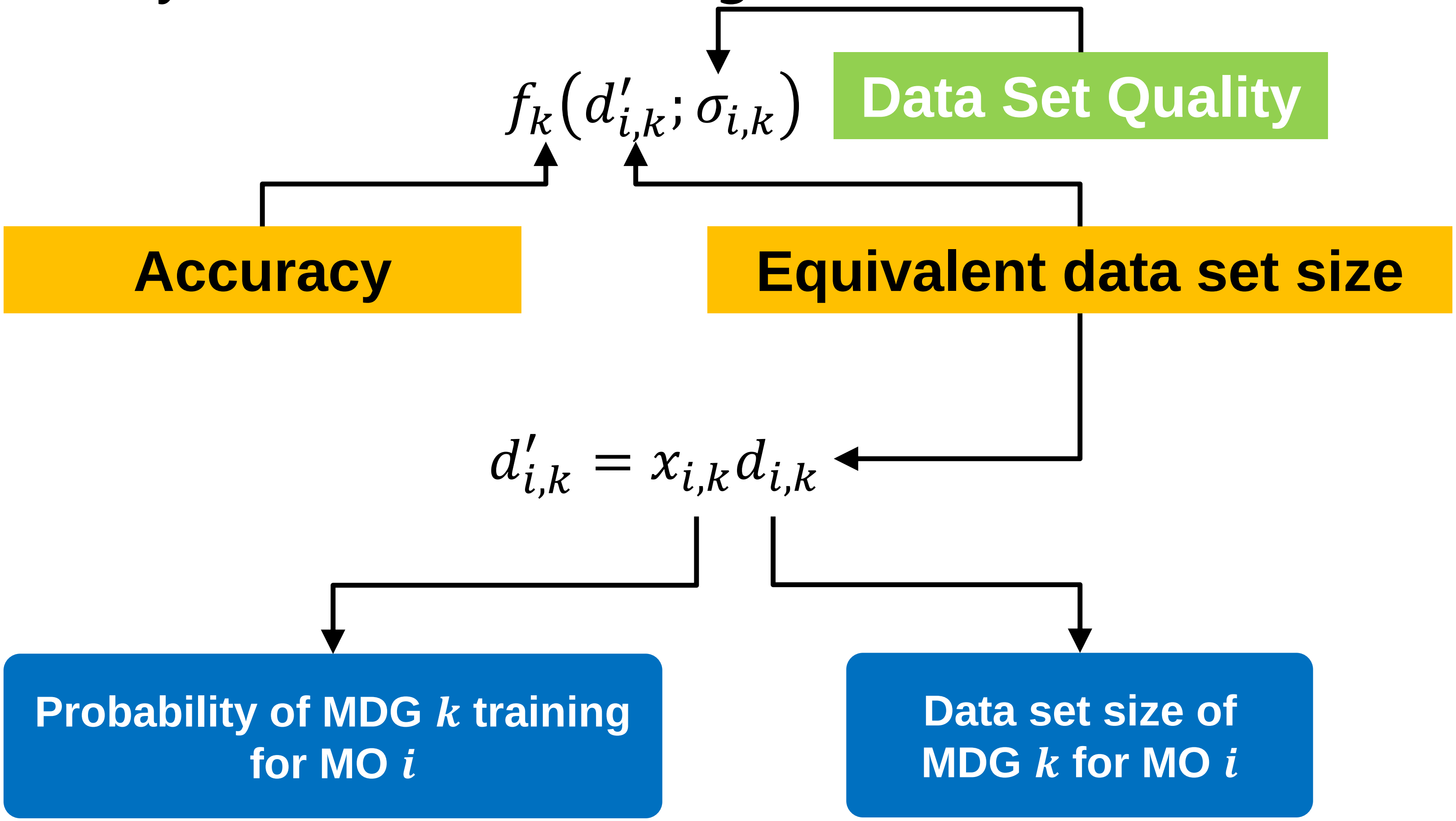


System Description

- Accuracy of machine learning model



System Description

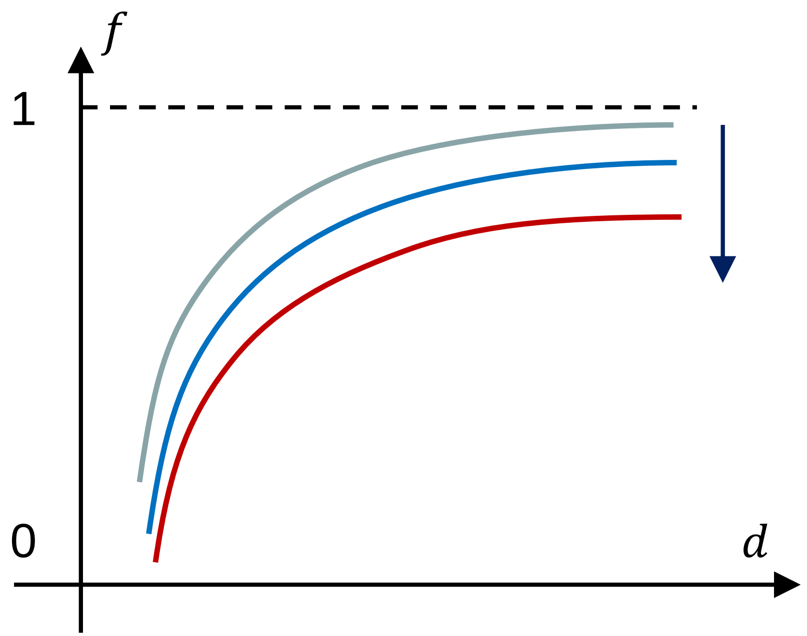
- Accuracy of machine learning model

$$f_k\left(d'_{i,k};\sigma_{i,k}\right)$$

- Properties of $f(d;\sigma)$

  Given σ
  - Non-decreasing
  - Concave
  - Bounded

  Given d
  - Decreasing with σ



System Description

- Utility of MO i
  - **Benefit**

    $$\zeta_{i,k} f\left(d_{i,k}; \sigma_{i,k}\right)$$

    Benefit coef. ($\zeta_{i,k}$) — **Accuracy** ($f\left(d_{i,k}; \sigma_{i,k}\right)$)

  - **Cost**

    $$p_{i,k} x_{i,k} d_{i,k} + \frac{\alpha_k}{2}\left(\sum_i x_{i,k} d_{i,k}\right)^2$$

    Price ($p_{i,k} x_{i,k} d_{i,k}$) — **Congestion** ($\frac{\alpha_k}{2}\left(\sum_i x_{i,k} d_{i,k}\right)^2$)

  - **Utility**

$$u_{i,k} = \zeta_{i,k} f\left(d_{i,k}; \sigma_{i,k}\right) - p_{i,k} x_{i,k} d_{i,k} - \frac{\alpha_k}{2}\left(\sum_i x_{i,k} d_{i,k}\right)^2$$

System Description

- Profit of MDG k
  - **Benefit**

$$\sum_i p_{i,k} x_{i,k} d_{i,k}$$

  - **Cost**

$$\sum_i c_{i,k} x_{i,k} d_{i,k}$$

Cost coef.

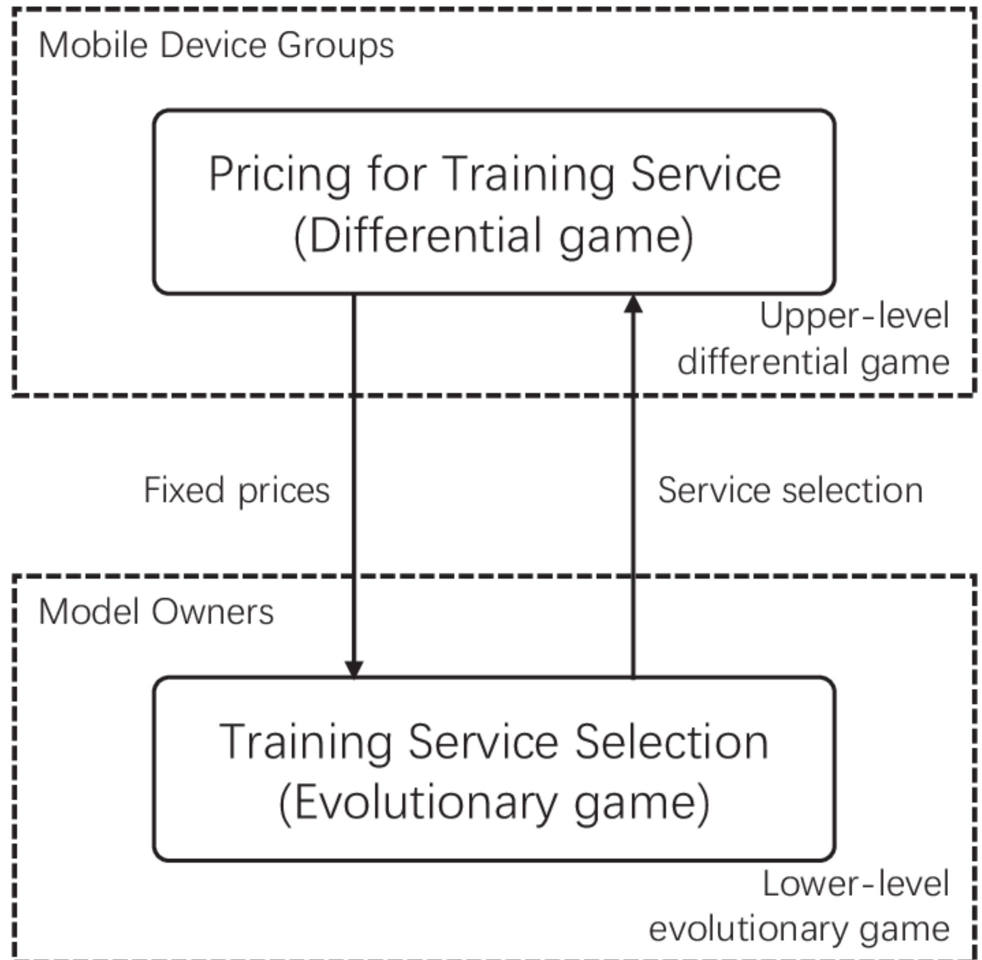
  - **Profit**

$$\Pi_k = \sum_i (p_{i,k} x_{i,k} d_{i,k} - c_{i,k} x_{i,k} d_{i,k})$$

Dynamic Games Formulation

Two-Layer Heretical Game

- Lower level: MOs
  - Evolutionary Game
- Upper level: MDGs
  - Differential Game



Dynamic Games Formulation

■ Lower Level Game

- **Replicator dynamics**

$$\dot{x}_{i,k}(t) = x_{i,k}(t)\left(u_{i,k}(t) - \bar{u}_i(t)\right) \cdots\cdots (1)$$

Utility of MO *i* selecting MDG *k*

MO *i*'s average utility

$$\bar{u}_i(t) = \sum_{i=1}^{K} x_{i,k}(t) u_{i,k}(t)$$

Dynamic Games Formulation

■ Upper Level Game

- **Optimal Control Problem(OCP)**

$$\max_{p_k(t)} \int_0^T \Pi_k(t)\mathrm{d}t$$

Accumulative Profit

$$\text{s.t. } \dot{x}_{i,k}(t) = x_{i,k}(t)\left(u_{i,k}(t) - \bar{u}_i(t)\right)$$

Lower Level Adaptation

$$\mathbf{x}_i(0) = \mathbf{x}_i^{(0)}, i \in \mathcal{N}$$

Initial Cond.

Dynamic Games Formulation

- Equilibrium Analysis (Lower Level)
  - *Definition*: The evolutionary equilibrium is the solution of the game defined in (1), i.e., replicator dynamics.
  - *Uniqueness of the equilibrium*

    Proved via *Cauchy- Lipschitz theorem*.
  - *Stability of the equilibrium*

    Proved via *Lyapunov's second method.*

Dynamic Games Formulation

■ Equilibrium Analysis (Upper Level)

- Solving the upper-level differential game is equivalent to solve K optimal control problems.
- The solution of OCP is further equivalent to maximize its corresponding Hamilton, defined as follows:

$$\max_{p_k(t)} \mathcal{H}_k(\mathbf{p}_k, \mathbf{p}_{-k}, \boldsymbol{\lambda}, \mathbf{x}) = \Pi_k(\mathbf{p}_k) + \sum_i \sum_k \lambda_{i,k} \dot{x}_{i,k}$$

$$\text{s.t.}\, x_i(0) = x_i^{(0)}, \lambda_{i,k}(T) = 0$$

Numerical Evaluation

Parameter	Setting
K	3
N	2
σ_i	[0.1, 0.15, 0.2]
$d_{i,k}$	4000
$\zeta_{i,k}$	6

Numerical Evaluation

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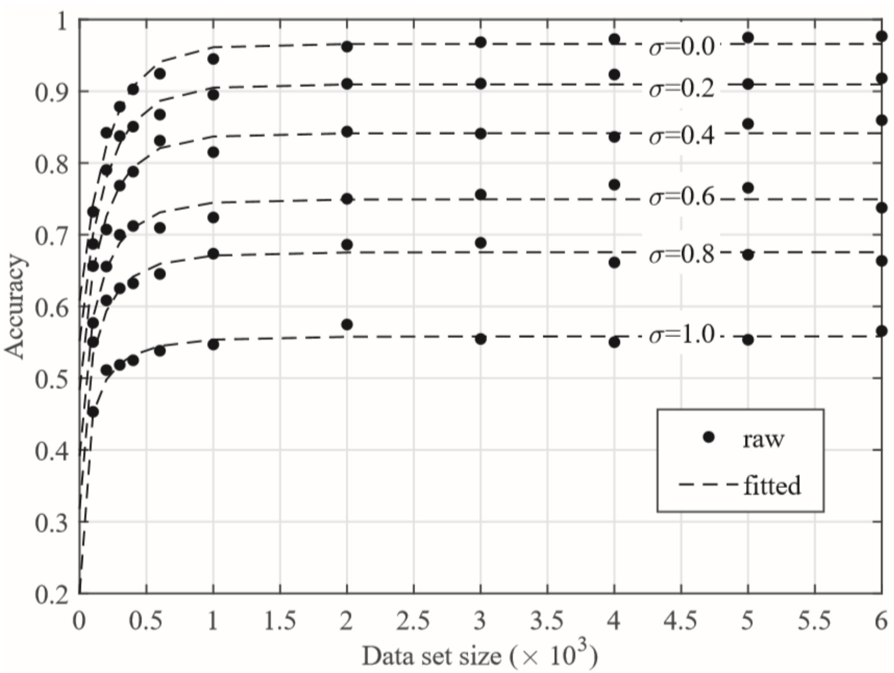
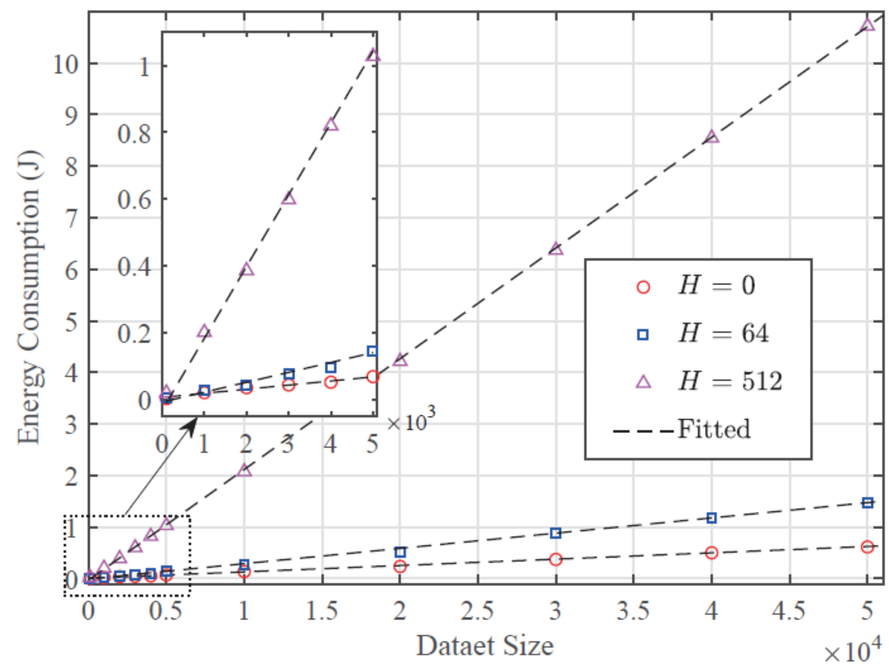


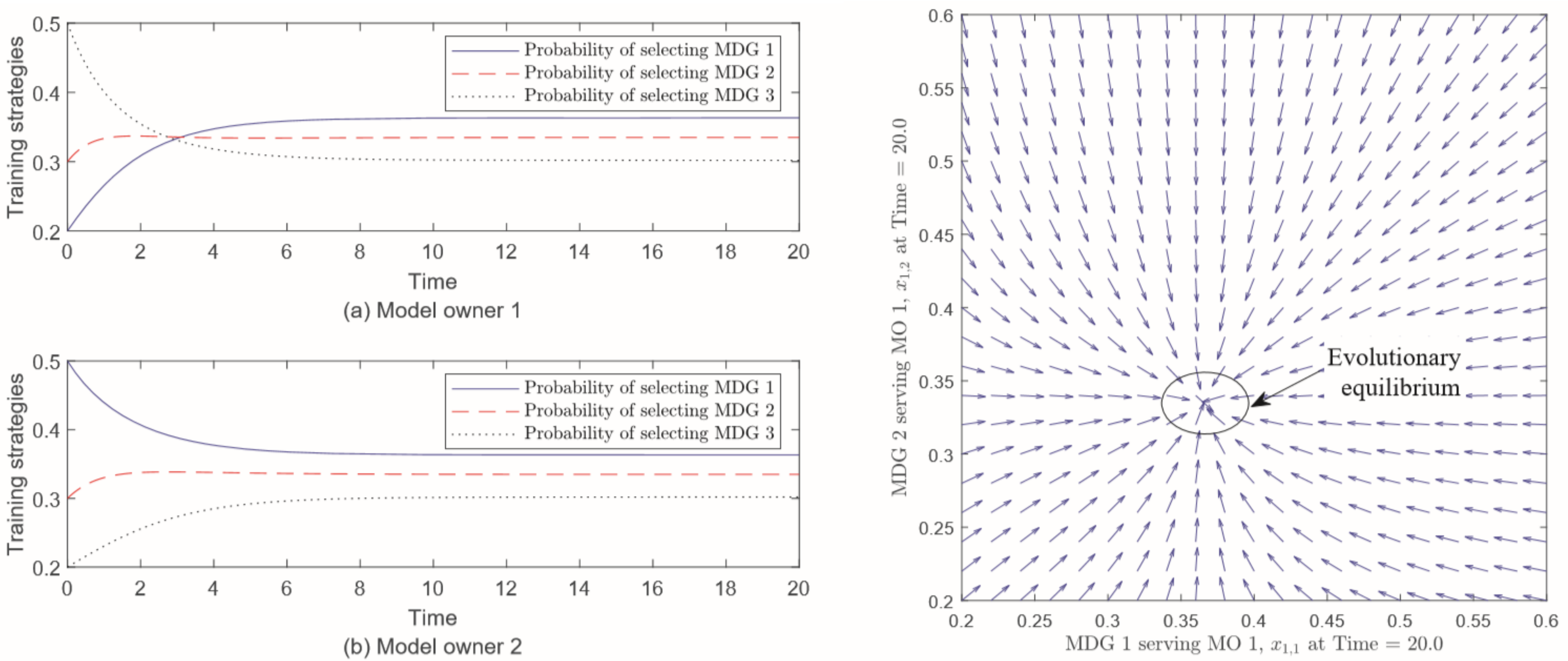
(a) Accuracy fitting



(b) Energy Consumption

Accuracy and Energy Consumption Fitting

Numerical Evaluation



(a) Evolution trajectories (b) Direction field

Evolution trajectories of MOs' selections and direction field of the replicator dynamics

Numerical Evaluation

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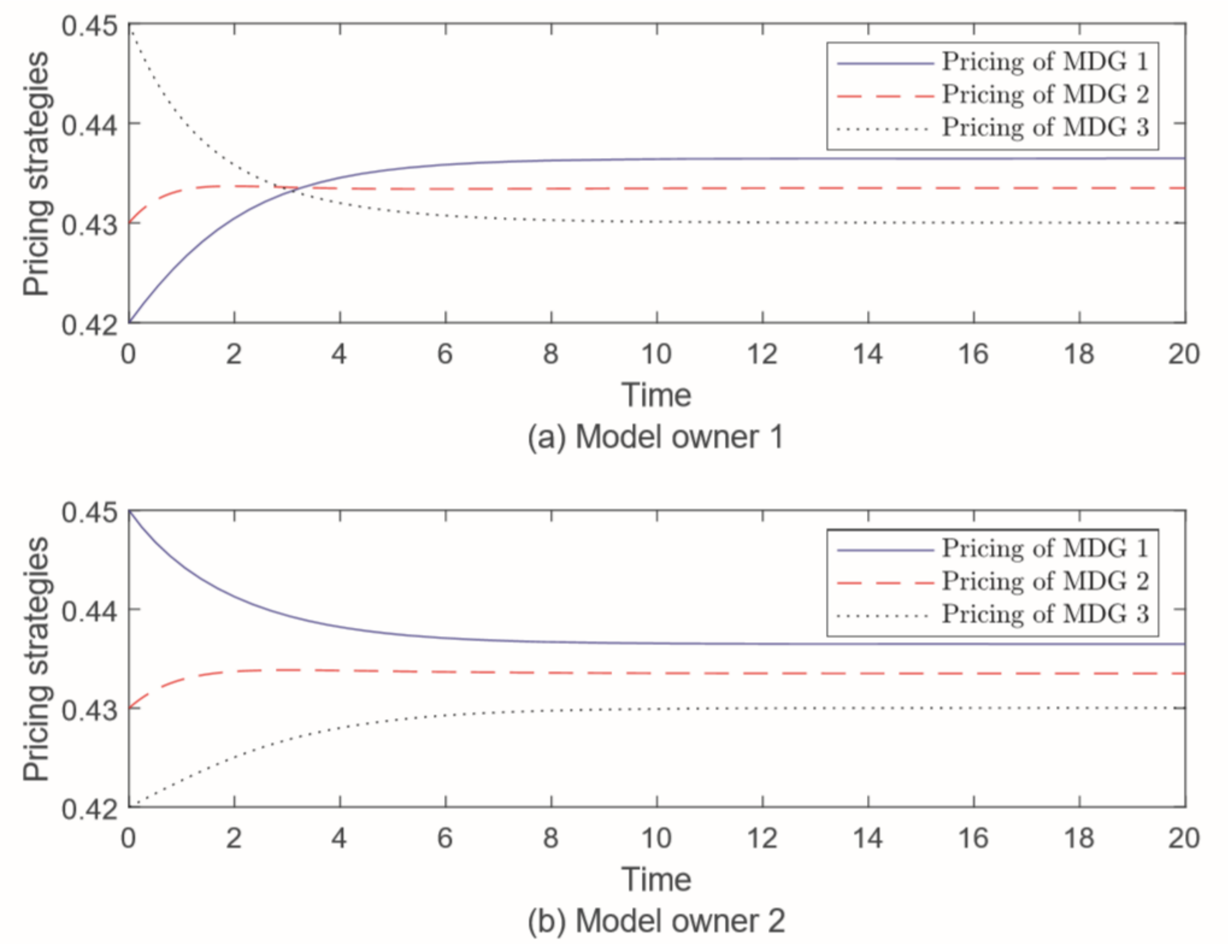

Pricing strategies of MDGs

Conclusions

- **Contributions**
  - ✓ We devise a two-layer dynamic game model consists of the lower-level evolutionary game of the model owners and the upper-level differential game of mobile device groups.
  - ✓ The solutions of the proposed two-layer dynamic game are analyzed theoretically and verified via numerical evaluations.
- **Future work:**
  - Devise more realistic solution for the game, such as deep reinforcement learning based method.

Questions & Answers

Thank you !